# Short Papers

## A Frequency-Varying Method for Simultaneous Measurement of Complex Permittivity and Permeability with an Open-Ended Coaxial Probe

Shoujun Wang, Maode Niu, and Deming Xu

***Abstract*— To measure the complex permittivity and permeability of materials simultaneously with an open-ended coaxial probe, one needs at least two independent reflections. Based on the fact that frequency is an independent variable for the probe's reflection coefficient, a new concept, namely the frequency-varying method (FVM), which achieves the independent reflections via changing frequency, has been proposed. Since the electromagnetic (EM) properties of materials themselves are functions of frequency, the FVM introduces interpolation techniques into the process of extracting EM parameters from multiple reflection coefficients. The successful experimental results on radar-absorbing coatings show the feasibility and good prospects of the FVM for characterizing EM properties of materials *in situ*. Compared with the thickness-varying method (TVM), which makes two measurements with two samples of different thicknesses, the FVM needs only one frequency-swept reflection measurement, thus simplifying and speeding up the measurement process, and improving accuracy and repeatability. Furthermore, the FVM has the ready capability to be extended to multiple-parameter measurements, and we may also find potential applications in other fields.**

***Index Terms*—Frequency-varying method, open-ended coaxial probe, permeability, permittivity.**

### I. Introduction

Open-ended coaxial probes have been investigated by many researchers for nondestructive characterization of dielectric materials [1]–[12]. However, most of these studies are limited to measuring the complex permittivity only. In general, the properties of materials are needed to be characterized by both the complex permittivity $\varepsilon$ and permeability $\mu$. Thus, it is necessary to extend the open-ended coaxial probe technique to measure $\varepsilon$ and $\mu$ simultaneously. To accomplish this target, one needs as least two independent reflections. The so-called thickness-varying method (TVM), which makes two reflection measurements with two samples of different thicknesses, has been proposed with either an open-ended coaxial probe [11], [12] or a rectangular-waveguide sensor [13]. Nevertheless, the TVM needs two mechanical operations; therefore, it is not convenient for measuring radar-absorbing coatings *in situ*, and error factors are introduced/increased in the measurement process.

In this paper, a new concept, namely, the frequency-varying method (FVM) has been proposed. The distinct advantage of the FVM is that it needs only one frequency-swept reflection measurement, thus simplifying and speeding up the measurement process, and improving accuracy and repeatability. Experimental results on radar-absorbing coatings are given to confirm the feasibility of the FVM.

### II. FVM

The open-ended sensor for nondestructive testing may be a coaxial line, rectangular, or circular waveguide. The material-under-test may be a semi-infinite/half-space sample, a one- or multiple-layer sample backed by a short or open circuit (air). As an example, here we consider the case of characterizing a microwave-absorbing coating using a flanged open-ended coaxial probe [see Fig. 1(a) and (b)]. The probe is pressed against the material-under-test, then with either an automatic network analyzer (ANA) or a six-port reflectometer, the reflection coefficient is measured and used to determine the electromagnetic (EM) parameters of the sample. The measurement system can be divided into two blocks [see Fig. 1(c)]. One is the hardware-oriented conversion, aimed at transferring the desired information $\varepsilon$ and $\mu$ into the raw result of measurement $(\Gamma)$. The other is the software-oriented reconstruction, aimed at extracting the desired information ($\varepsilon$ and $\mu$) from the raw result of conversion $(\Gamma)$. Once the field-theory-based mathematical model relating the EM parameters with the reflection coefficient is constructed, the main problem left is parameter extraction. As is well known, the inverse problem is generally more difficult and time consuming since it may be ill conditioned and its solution has to rely on iterative optimization techniques.

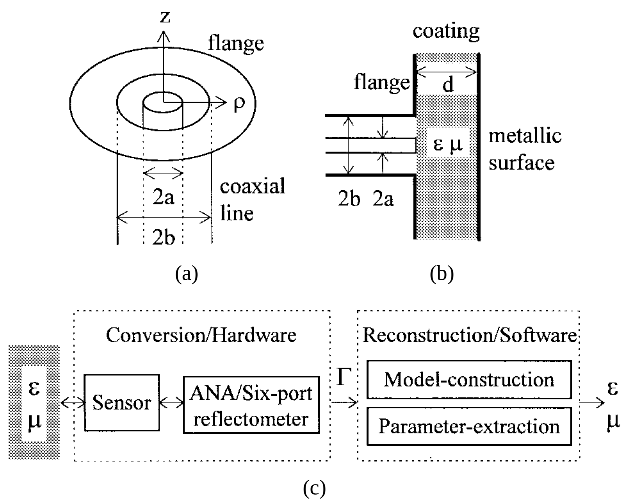


Fig. 1. (a) Geometry of a flanged open-ended coaxial probe. (b) Configuration for characterizing microwave-absorbing coatings. (c) Measurement system.

To measure the complex permittivity and permeability simultaneously with an open-ended probe, one needs at least two independent reflections. The reflection coefficient $\Gamma(a, b, f, \varepsilon(f), \mu(f), d)$ is a complicated function of the coaxial dimensions $a$ and $b$, frequency $f$, EM parameters $\varepsilon$ and $\mu$, and sample thickness $d$. One way to produce the needed two independent reflections is the so-called TVM. In this paper, a new concept, namely the FVM, achieves the needed independent reflection coefficients via changing frequency. It needs only one frequency-swept reflection measurement over the interested frequency range, then the remaining task is parameter extraction from the measured data. The FVM is based on the following two points. Firstly, frequency is an independent variable for the reflection coefficient, as can be seen from $\Gamma(a, b, f, \varepsilon(f), \mu(f), d)$, hence, the reflection coefficient changes with frequency. Secondly, since the EM properties of materials $\varepsilon(f)$ and $\mu(f)$ themselves are functions of frequency, the interpolation techniques are introduced into the process of extracting $\varepsilon$ and $\mu$ from the frequency-swept reflection data.

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The polynomial interpolation techniques have been addressed. The simplest interpolation approximation

$$\varepsilon(f) \approx \varepsilon(f + \Delta f), \mu(f) \approx \mu(f + \Delta f) \quad (1)$$

needs the least two frequency points/reflection coefficients to reconstruct the complex permittivity and permeability, and the results are interpreted as an average of the actual values over $\Delta f$. The linear interpolation

$$\varepsilon(f) = af + b \quad \mu(f) = cf + d \quad (2)$$

extracts the EM parameters from four frequency points needed to determine four complex constants $a$, $b$, $c$, $d$. Similarly, the parabolic interpolation which extracts the EM parameters from six frequency points has to determine six complex constants. When implementing the FVM, the broad-band frequency-swept reflection coefficients are first measured, then the complex permittivity and permeability are reconstructed from the multiple reflection coefficients according to the adopted interpolation technique, and the extracted parameters represent an average of the actual values over these frequency points; this process continues throughout the entire frequency range.

The choice of the order of the interpolation and frequency interval $\Delta f$ (usually equal, but not always) depends on many factors such as the natural characteristics of the material's EM properties and requirements of measurement speed and accuracy. In general, for more dispersive media and for more critical accuracy requirements, higher order interpolation should be used. At the same time, the inverse problem will become more difficult and time consuming with the ever-increasing variables to be determined. A tradeoff between accuracy and speed must be made in practical real-time and on-site measurements. We have found that for a large variety of practical solid materials, their EM properties vary slowly with frequency, and the simplest interpolation approximation (1) is adequate. The frequency interval should guarantee that the reflection coefficient is changed enough to be distinguished by an ANA or a six-port reflectometer. To improve accuracy, it should be selected as small as possible, especially for the dispersive materials whose $\varepsilon(f)$ and $\mu(f)$ vary rapidly with frequency. The appropriate frequency interval may be determined by rule-of-thumb or by sensitivity analysis and numerical simulation.

## III. Experimental Verification

An air-filled coaxial-line probe with a flange at its open end was fabricated. Its inner and outer radii are $a = 1.52$ mm and $b = 3.47$ mm, respectively, and its cutoff frequency of the likely excited higher order mode $TE_{11}$ is 19.124 GHz. The broad-band frequency-swept reflection coefficients are measured using an ANA (HP 8510B). The widely used short-circuit, open-circuit, and reference liquid standards are employed to calibrate the coaxial probe based on the cross-ratio transformation [3], [6], [7]

$$\frac{(\rho - \rho_1)(\rho_i - \rho_3)}{(\rho - \rho_3)(\rho_2 - \rho_1)} = \frac{(\Gamma - \Gamma_1)(\Gamma_2 - \Gamma_3)}{(\Gamma - \Gamma_3)(\Gamma_2 - \Gamma_1)} \quad (3)$$

where $\rho$ and $\rho_i$ $(i = 1, 2, 3)$ are the measured reflection coefficients of the material-under-test and the three standards, respectively, and $\Gamma$ and $\Gamma_i$ are the corresponding theoretical values. The following quasi-static admittance model, which takes into account only the dominant TEM mode, is used in calibration and parameter extraction:

$$Y = \frac{1-\Gamma}{1+\Gamma} = \frac{jk_0\varepsilon}{\ln\dfrac{b}{a}} \int_0^\infty \frac{\coth(d\sqrt{\lambda^2 - k_0^2\varepsilon\mu})}{\lambda\sqrt{\lambda^2 - k_0^2\varepsilon\mu}} \cdot [J_0(\lambda a) - J_0(\lambda b)]^2 \, d\lambda. \quad (4)$$

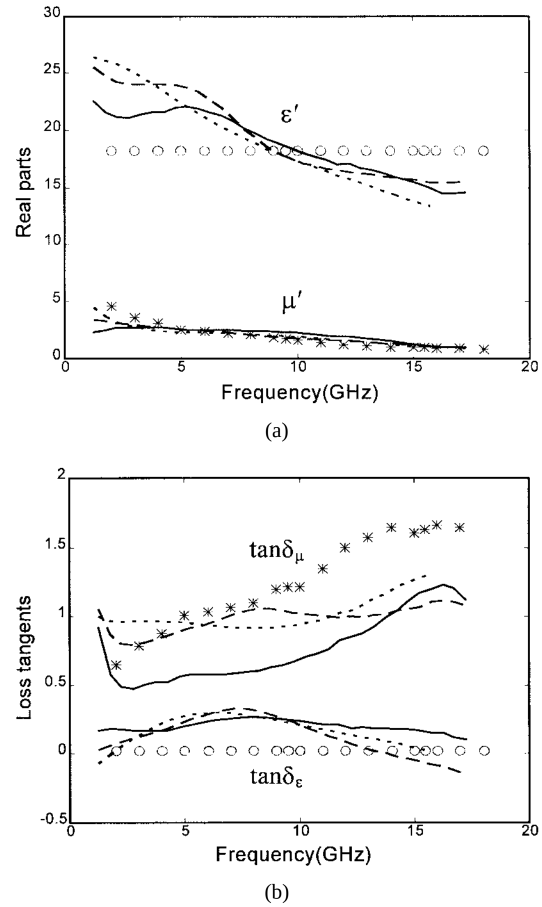


Fig. 2. The FVM results of 9052, $d = 1.22$ mm, calibrated with short–open–water (solid), short–open–methanol (dotted), and short–water–methanol (dashed), $\Delta f = 0.5$ GHz. "o" and "$*$" denote the permittivity and permeability provided by the fabricator, respectively. (a) Real parts. (b) Loss tangents.

An iterative algorithm for multidimensional optimization was coded to reconstruct $\varepsilon$ and $\mu$ from the corrected reflection coefficients using the simplest interpolation approximation.

Measurement on several radar-absorbing materials (RAM's) was conducted. Fig. 2 shows the FVM results of the RAM coating 9052 (ferrite absorber) calibrated with different standards. On the whole, the measured results are in agreement with the available reference data, indicating the soundness of the FVM. The influence of the calibration standards is also obvious. It is difficult to exactly evaluate different standards, but from the measured results of $\tan\ \delta_\mu$, we can see that methanol is better than water for the case here. The explanation is that methanol has a reflection coefficient similar to that of the material-under-test [6], [7]. The measurement sensitivity and uncertainty are strongly dependent on the sample thickness. As illustrated in Fig. 3, the inconsistency of the FVM results of MF116 with different thicknesses is significant. One observes a closer agreement for the real parts, especially for $\mu'$, this is because the material-under-test is in strong magnetic fields with a short-circuit backing. On the contrary, the loss tangents show a larger discrepancy. We have found that the numerical convergence is sensitive to both the sample thickness and reflection-coefficient measurement error. For a fixed thickness sample, the stable convergence may be branched off over some frequency range and exhibits resonant-like phenomena. In addition, the reflection measurement technique has the inherent limitations that it is difficult to obtain reasonably accurate loss tangents for low-loss materials $(\tan\delta{<}0.1)$. Research work such

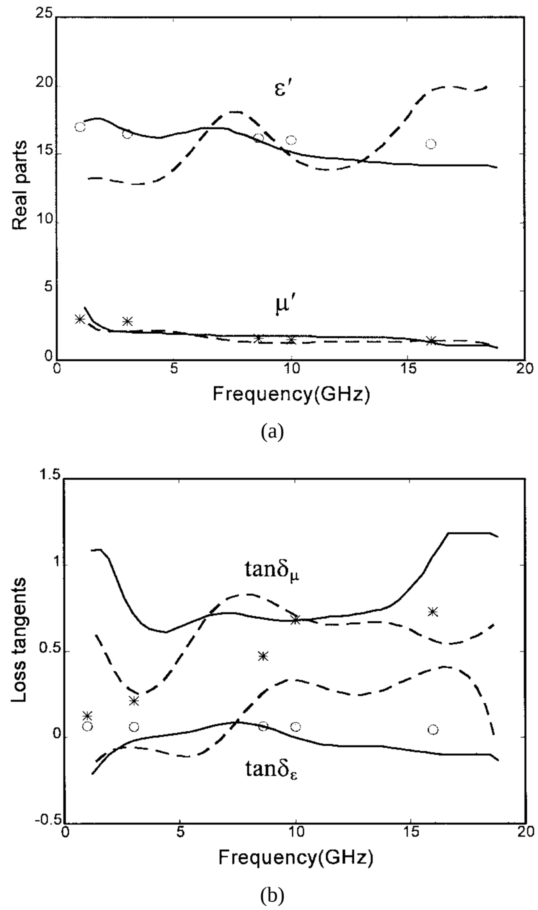


Fig. 3. The FVM results of MF116 with $d = 0.7$ mm (dashed), $d = 1.4$ mm (solid), calibrated with short–methanol–water, $\Delta f = 0.36$ GHz. "o" and "$*$" denote the permittivity and permeability provided by the fabricator, respectively. (a) Real parts. (b) Loss tangents.

as error analysis and calibration-technique improvement should be investigated in the future.

## IV. Conclusions

Based on the facts that frequency is an independent variable for the reflection coefficient and EM properties of materials themselves are functions of frequency, the FVM, which employs the frequency-sweep and interpolation techniques to simultaneously determine permittivity and permeability, has been suggested in this paper. The successful experimental results on microwave-absorbing coatings illustrate the feasibility and good prospects of the FVM for characterizing EM properties of materials *in situ*. While the TVM necessitates two mechanical operations, the FVM proposed here needs only one frequency-swept reflection measurement, and thus, simplifies and speeds up the measurement process and improves accuracy and repeatability. Furthermore, the FVM has the ready capability to be extended to multiple-parameter measurements such as determining permittivity, permeability, and thickness simultaneously, and we may find potential applications in other fields.

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